



Fortify Sample Exam 1A

MATHEMATICAL METHODS

Written examination 1

Reading time: 15 minutes

Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

| <i>Number of questions</i> | <i>Number of questions to be answered</i> | <i>Number of marks</i> |
|----------------------------|---|------------------------|
| 10 | 10 | 40 |

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied

- Question and answer booklet of 15 pages.
- Formula sheet.
- Working space is provided throughout the book.

Instructions

- Write your **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination

- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the space provided.

In all questions where a numerical answer is required, an exact value must be given, unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (4 marks)

a. Differentiate $x \log_e(x^2)$ with respect to x .

2 marks

b. Let $h(x) = 2e^{\sin(3x)}$.

Evaluate $h' \left(\frac{\pi}{18} \right)$.

2 marks

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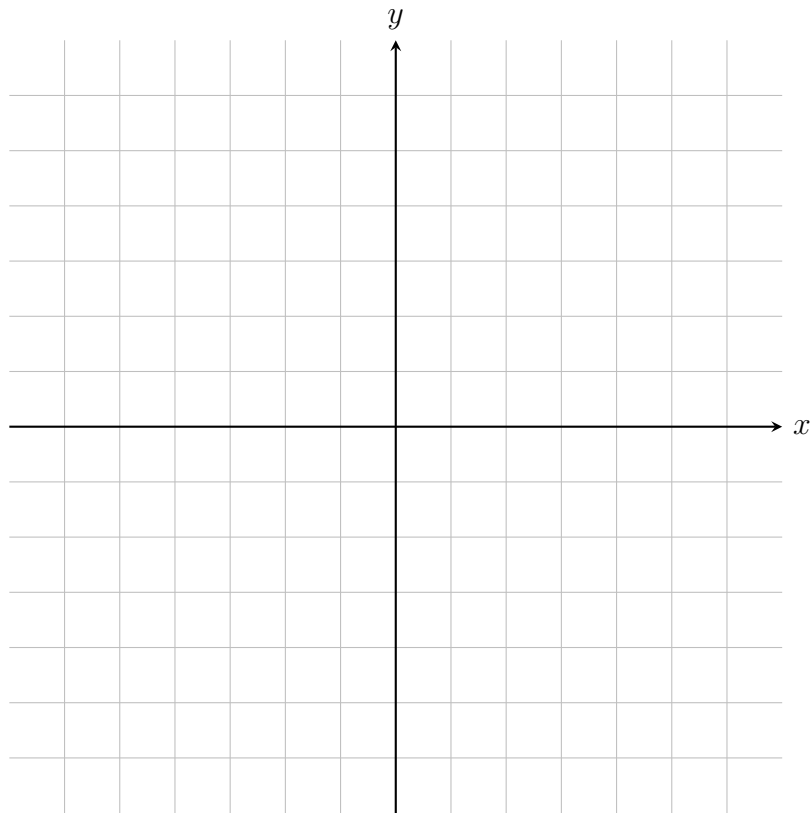
Question 2 (3 marks)

The line $0 = y - bx - 4 + \frac{\pi}{2\sqrt{3}}$ is a tangent to the curve $y = 2 \sin\left(\frac{x}{2}\right) + a$ at the point $\left(\frac{\pi}{3}, c\right)$, where a , b and c are real constants. Find the values of a , b and c .

Question 3 (3 marks)

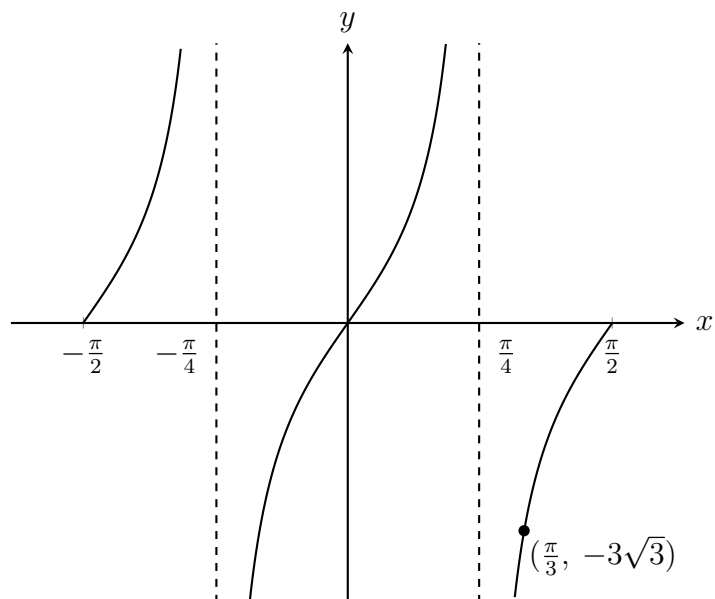
On the axes below, sketch the graph of $f(x) = 4x^2 - 2x^3$, $x \in [-1, 2]$.

Label all endpoints and axis intercepts.



Question 4 (2 marks)

A section of the graph of f is shown below.



If the function f is defined by $f(x) = a \tan(bx)$, find the values of a and b .

Question 5 (3 marks)

Each night, Jack gets takeaway for dinner. He can choose from a healthy dinner or an unhealthy dinner. If he has a healthy meal one night, the probability that he has a healthy meal the next night is 0.7. If he has an unhealthy meal one night, the probability that he has an unhealthy meal the following night is 0.2. Jack has a healthy meal on a Sunday.

What is the probability that he has a healthy meal on the following Tuesday night?

Question 6 (5 marks)

The function

$$f(x) = \begin{cases} a \cos\left(\frac{\pi x}{2}\right), & 0 \leq x \leq 1 \\ 0, & \text{elsewhere} \end{cases}$$

is a probability density function for the continuous random variable X .**a.** Show that $a = \frac{\pi}{2}$.

2 marks

b. Find $\Pr\left(X \geq \frac{1}{2} \mid X \geq \frac{1}{3}\right)$.

3 marks

Question 7 (3 marks)

Let $f : \mathbb{R}^- \rightarrow \mathbb{R}$, $f(x) = \frac{2}{x}$.

a. Find h , where $h(x) = f(f(2x))$, and state the maximal domain for which h is defined.

2 marks

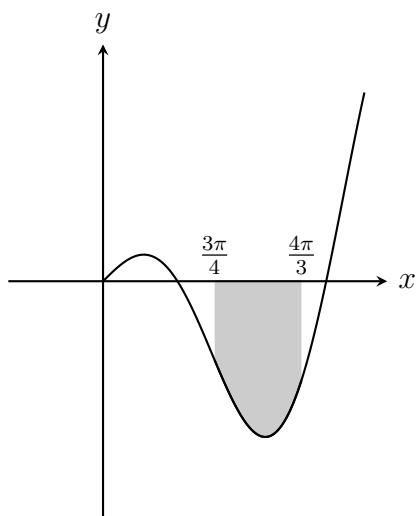
b. Evaluate $h^{-1}(24)$, where h^{-1} is the inverse function of h .

1 mark

TURN OVER

Question 8 (4 marks)

Part of the graph of the function $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = x \cos(x)$ is shown below.



a. Find the derivative of $2x \sin(x)$.

1 mark

b. Hence, find the area of the shaded region in the diagram above.

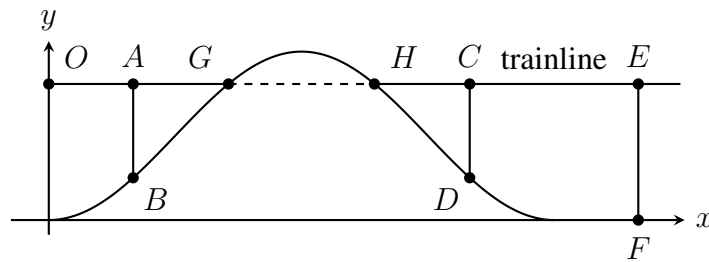
3 marks

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Question 9 (7 marks)

A trainline runs along a bridge, through a tunnel in a mountain, and emerges onto another bridge on the other side of the mountain as shown in the diagram below.



The shape of the mountain is modelled by the function

$$y = 40 \sin \left(\frac{\pi}{60}(x - 30) \right) + 40$$

The length of tunnel GH is 20 metres and the sum of the lengths of bridge supports AB and CD is $(40\sqrt{30} + 40)$ metres. The distance from A to G is equal to the distance from H to C .

a. How high above ground level is the peak of the mountain?

1 mark

b. What is the length of bridge support EF ?

3 marks

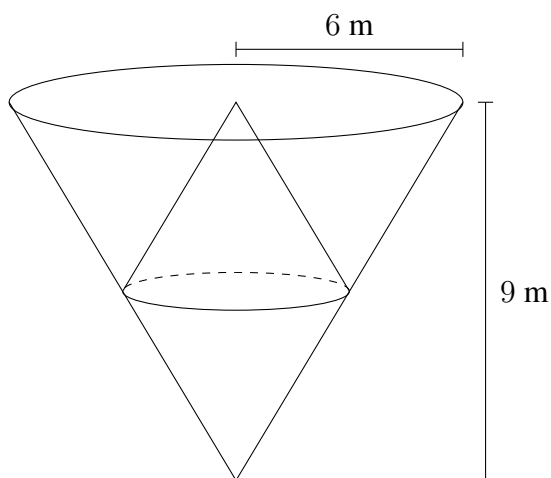
c. If a train travels at 50 metres per second, how long does it take to travel from A to C ?

3 marks

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Question 10 (6 marks)

A big inverted right circular cone holds a smaller right circular cone inside it as shown in the diagram below.



The radius of the bigger cone is 6 m and the height is 9 m. The radius of the smaller cone is r m and the height is h m.

a. Find h in terms of r .

2 marks

b. Find V , the volume of the smaller cone, in terms of r .

1 mark

c. Find the value of r for which V is at a maximum and state this maximum volume.

3 marks

END OF QUESTION AND ANSWER BOOK



MATHEMATICAL METHODS

Written examination 1

FORMULA SHEET

Instructions

This formula sheet is provided for your reference.
A questions and answer book is provided with this formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Formula Sheet

Mensuration

| | | | |
|-----------------------------------|------------------------|---------------------|-------------------------|
| area of a trapezium | $\frac{1}{2}(a + b)h$ | volume of a pyramid | $\frac{1}{3}Ah$ |
| curved surface area of a cylinder | $2\pi rh$ | volume of a sphere | $\frac{4}{3}\pi r^3$ |
| volume of a cylinder | $\pi r^2 h$ | area of a triangle | $\frac{1}{2}bc \sin(A)$ |
| volume of a cone | $\frac{1}{3}\pi r^2 h$ | | |

Calculus

| | |
|--|--|
| $\frac{d}{dx}(x^n) = nx^{n-1}$ | $\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$ |
| $\frac{d}{dx}((ax + b)^n) = an(ax + b)^{n-1}$ | $\int (ax + b)^n dx = \frac{1}{a(n+1)}(ax + b)^{n+1} + c, n \neq -1$ |
| $\frac{d}{dx}(e^{ax}) = ae^{ax}$ | $\int e^{ax} dx = \frac{1}{a}e^{ax} + c$ |
| $\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$ | $\int \frac{1}{x} dx = \log_e(x) + c, x > 0$ |
| $\frac{d}{dx}(\sin(ax)) = a \cos(ax)$ | $\int \sin(ax) dx = -\frac{1}{a} \cos(ax) + c$ |
| $\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$ | $\int \cos(ax) dx = \frac{1}{a} \sin(ax) + c$ |
| $\frac{d}{dx}(\tan(ax)) = \frac{a}{\cos^2(ax)} = a \sec^2(ax)$ | |
| product rule | $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$ |
| quotient rule | $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$ |
| chain rule | $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ |

Formula Sheet

Probability

| | | | |
|---|--------------|---|--|
| $\Pr(A) = 1 - \Pr(A')$ | | $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$ | |
| $\Pr(A B) = \frac{\Pr(A \cap B)}{\Pr(B)}$ | | | |
| mean | $\mu = E(X)$ | variance | $\text{var}(X) = \sigma^2 = E((X - \mu)^2) = E(X^2) - \mu^2$ |

| Probability distribution | | Mean | Variance |
|--------------------------|-------------------------------------|---|--|
| discrete | $\Pr(X = x) = p(x)$ | $\mu = \sum x p(x)$ | $\sigma^2 = \sum (x - \mu)^2 p(x)$ |
| continuous | $\Pr(a < X < b) = \int_a^b f(x) dx$ | $\mu = \int_{-\infty}^{\infty} x f(x) dx$ | $\sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$ |

Sample proportions

| | | |
|-------------------------|--|--|
| $\hat{P} = \frac{X}{n}$ | mean | $E(\hat{P}) = p$ |
| standard deviation | $\text{sd}(\hat{P}) = \sqrt{\frac{p(1-p)}{n}}$ | approximate confidence interval $\left(\hat{p} - z\sqrt{\frac{p(1-p)}{n}}, \hat{p} + z\sqrt{\frac{p(1-p)}{n}} \right)$ |

END OF FORMULA SHEET