TRIUMPH
PUBLICATIONS

Fortify Sample Exam 2A

FURTHER MATHEMATICS

Written examination 2

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

QUESTION AND ANSWER BOOK

Structure of book

Section A – Core	*Number of questions*	*Number of questions to be answered*	*Number of marks*
	7	7	36
Section B – Modules	*Number of modules*	*Number of modules to be answered*	*Number of marks*
	4	2	24
			Total 60

- Students are to write in blue or black pen.
- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 37 pages.
- Formula sheet
- Working space is provided throughout the book.

Instructions
- Write your **name** and **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION A - Core

Instructions for Section A

Answer **all** questions in the spaces provided.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.

In 'Recursion and financial modelling', all answers should be rounded to the nearest cent unless otherwise instructed.

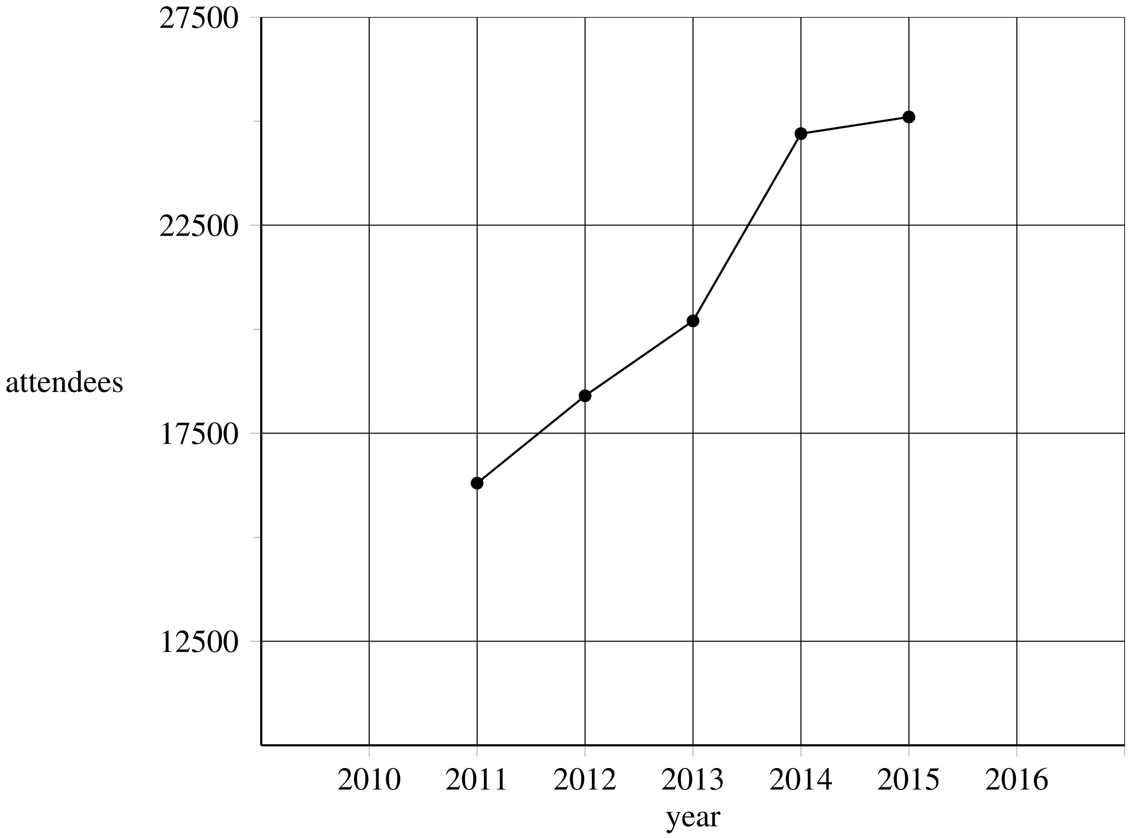
Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Data Analysis

Question 1 (6 marks)

The table below shows the number of attendees at an Australian music festival over yearly intervals for the period 2010 to 2016.

Year	2010	2011	2012	2013	2014	2015	2016
Attendees	14,700	16,300	18,400	20,200	24,700	25,100	24,900



a. Complete the time series plot above by plotting the number of attendees for the years 2010 and 2016. 1 mark

b. Briefly describe the general trend in the data.

1 mark

In the table below, the variable *year* has been rescaled using $2010 = 0$, $2011 = 1$ and so on. The new variable is *time*.

Year	2010	2011	2012	2013	2014	2015	2016
Time	0	1	2	3	4	5	6
Attendees	14,700	16,300	18,400	20,200	24,700	25,100	24,900

c. Use the variables *time* and *attendees* to write down the equation of the least squares regression line that can be used to predict *attendees* from *time*. Take *time* as the explanatory variable.

Write your answer correct to the nearest whole numbers.

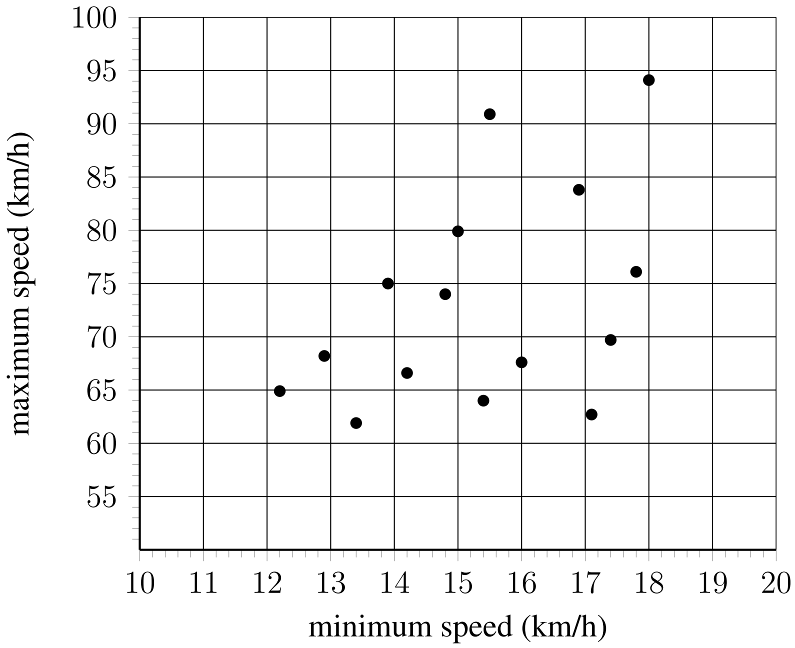
2 marks

d. In the year 2017, 26,300 people attended the music festival. Calculate the residual value for 2017 if the least squares regression line calculated in **part c.** is used to predict the number of attendees in 2017.

2 marks

Question 2 (7 marks)

The maximum and minimum speed at which a Year 12 student threw their textbooks across the room on each of the 15 days leading up to their exams was recorded in the scatterplot below.



The correlation coefficient for this set of data is $r = 0.444$.

The equation of the least squares regression line for this data set is

$$max\ speed = 35.6 + 2.45 \times min\ speed$$

a. Draw this least squares regression line on the scatterplot above.

1 mark

b. Interpret the y-intercept of the least squares regression line in terms of maximum speed and minimum speed.

1 mark

c. Describe the relationship between the maximum speed and the minimum speed in terms of strength and direction.

1 mark

d. Interpret the slope of the least squares regression line in terms of the maximum speed and minimum speed.

1 mark

e. Determine the percentage of variation in the maximum speed that may be explained by the variation in the minimum speed. Write your answer correct to the nearest whole percentage.

1 mark

On the day that the minimum speed was 14.8 km/h, the maximum speed was 74 km/h.

f. Determine the residual value for this day if the least squares regression line is used to predict maximum speed. Write your answer correct to one decimal place.

2 marks

Question 3 (6 marks)

The stem plot in Figure 1 shows the distribution of the average age, in years, at which men first enjoyed coffee in 15 towns.

Figure 1: average age in years at which men first enjoyed coffee

key: 18|5 = 18.5 years

16	0
17	0 6
18	3 7 9
19	3 7 7 9
20	1 6
21	5 8
22	1

a. For these towns, determine:

i. the highest average age of men when they first enjoyed coffee

1 mark

ii. the median average age of men when they first enjoyed coffee

1 mark

The stem plot in Figure 2 shows the distribution of the average age, in years, of women when they first enjoyed coffee in 15 towns.

Figure 2: average age in years at which women first enjoyed coffee

key: 17|3 = 17.3 years

16	7 8
17	0 3 3 6 8
18	0 0 4 8
19	0 5 6
20	
21	7

b. For these towns, determine the interquartile range (IQR) for the average age of women when they first enjoyed coffee

1 mark

If the data values displayed in Figure 2 were used to construct a boxplot with outliers, then the average age that women first enjoyed a coffee at 21.7 years old would be shown as an outlier.

c. Explain why this is so. Show an appropriate calculation to support your explanation.

3 marks

Question 4 (5 marks)

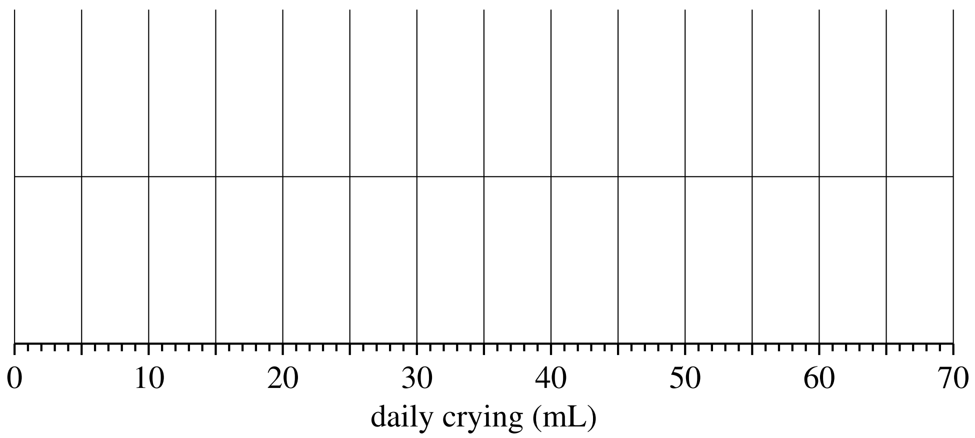
A student records how much they cry in the months leading up to their exams by measuring the daily volume of their tears in mL.

a. The five number summary for the distributions of daily crying for the month of August is displayed in the table below.

	Daily Crying (mL)
Minimum	10
Q_1	14
Median	35
Q_3	49
Maximum	68

There are no outliers in this distribution.

i. Use the five number summary above to construct a boxplot on the grid below.

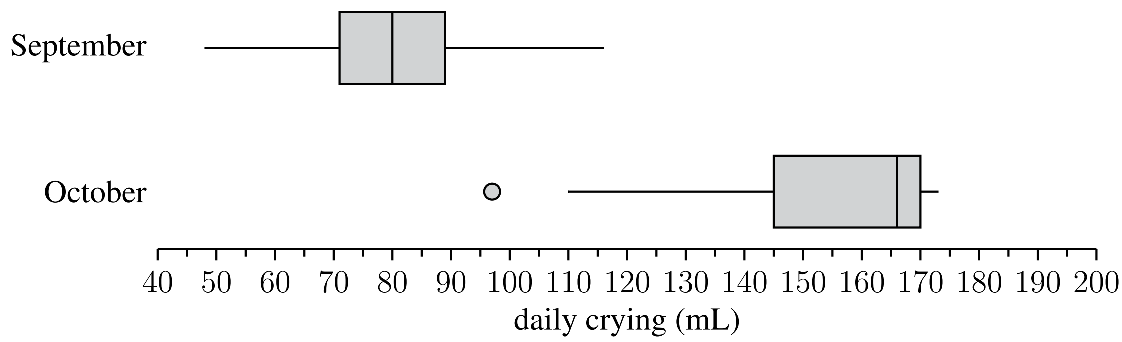


1 mark

ii. What percentage of days did the student cry 49mL or more of tears this particular August?

1 mark

b. The boxplots below display the distribution of daily crying for the months of September and October.



i. Describe the shapes of the distributions of daily crying (including outliers) for September and October.

September ______________________________

October ______________________________

1 mark

ii. Determine the value of the lower fence for the October boxplot.

____

____

____

____

1 mark

iii. Using the information from the boxplots, explain why 'daily crying' is associated with the month of the year. Quote the values of appropriate statistics in your response.

____

____

____

1 mark

Recursion and financial modelling

Question 5 (5 marks)

John is a professional 'player'. He uses a pair of rollerskates to help slide into people's DMs.

The value of John's rollerskates is depreciated using the **flat rate** method of depreciation. The value of John's rollerskates in dollars, V_n, after n years, can be modelled by the recurrence relation shown below.

$$V_0 = 235, \qquad V_{n+1} = V_n - 39.95$$

a. Recursion can be used to calculate the value of John's rollerskates after two years. Complete the calculations below by writing the appropriate numbers in the boxes provided.

$$V_0 = 235$$

$$V_1 = 235 - \boxed{} = \boxed{}$$

$$V_2 = \boxed{} - \boxed{} = 155.10$$

2 marks

b. **i.** By how many dollars is the value of John's rollerskates depreciated each year?

---

1 mark

ii. Calculate the annual **flat rate** of depreciation in the value of the rollerskates. Write your answer as a percentage.

---

1 mark

The value of John's rollerskates could also be depreciated using the **reducing balance** method of depreciation. The value of the rollerskates in dollars, R_n, after n years, can be modelled by the recurrence relation shown below.

$$R_0 = 235, \qquad R_{n+1} = 0.79R_n$$

c. At what annual percentage rate is the value of John's rollerskates depreciated each year?

---

1 mark

Question 6 (5 marks)

A young couple invests $16,250 at the beginning of a five year period. After five years of compounding interest, the value of the investment reaches $25,000.

a. How much interest was earned during the five years of this investment?

1 mark

Interest on the account had been calculated and paid monthly.

b. What was the annual rate of interest for this investment? Write your answer correct to one decimal place.

1 mark

c. The $25,000 was re-invested in another account for a year. The new account paid interest at a rate of 3.6% per annum, compounding quarterly. At the end of each quarter, the couple added an additional $420 to the investment.

i. The equation below can be used to determine the account balance at the end of the first quarter, immediately after the $420 was added.

Complete by filling in the boxes.

$$\text{Balance} = 25\ 000 \times \left(1 + \boxed{}\right) + \boxed{}$$

2 marks

ii. What was the account balance at the end of the year? Write your answer correct to the nearest dollar.

1 mark

Question 7 (2 marks)

Evan and Zoe decide to set up an annual university scholarship. To finance the scholarship, they invest in a perpetuity that pays \$18,252.45 annually at a rate of 4.7% per annum.

a. Determine the amount Evan and Zoe invested initially to fund the scholarship.

1 mark

b. When will Evan and Zoe have to invest more money to keep their scholarship going?

1 mark

END OF SECTION A

SECTION B - Modules

Instructions for Section B

Select **two** modules and answer **all** questions within the selected modules.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Contents **Page**

Module 1 - Matrices

Question 1 (3 marks)

A group of burglars steal a safe full of cash.

Matrix W contains the number of each type of note.

$$W = \begin{array}{c} \begin{bmatrix} 71 \\ 36 \\ 74 \\ 47 \end{bmatrix} \end{array} \begin{array}{c} A \\ B \\ C \\ D \end{array}$$

a. Write down the order of matrix W.

1 mark

b. Matrix V contains the value of each of the notes.

$$V = \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 5 & 10 & 20 & 50 \end{bmatrix} \end{array}$$

i. Calculate the matrix product $Y = V \times W$.

1 mark

ii. What does matrix Y represent?

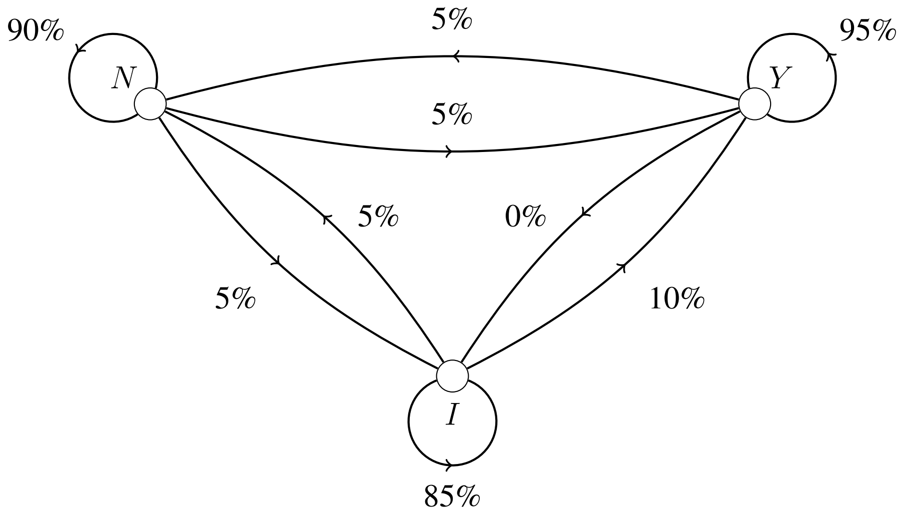
1 mark

Question 2 (5 marks)

Each month, Gotham Council holds a survey to determine citizen approval numbers when it comes to Batman roaming the streets of Gotham City as a vigilante. Past surveys have shown that approval can change month to month.

Citizens have three options in this survey: Yes to Batman roaming the streets (Y), No to Batman roaming the streets (N) and Indifferent to Batman roaming the streets (I).

a. The transition diagram below shows the percentage of citizens who are expected to change their approval from month to month.



i. Of the citizens who voted 'Yes' this month, what percentage are expected to vote 'No' next month?

1 mark

ii. Of the citizens who were 'Indifferent' this month, what percentage will no longer be 'Indifferent' next month?

1 mark

In January 2018, 145,000 citizens were surveyed. The number of citizens surveyed each month is to remain constant. The state matrix that indicates the number of citizens who are expected to have an approval preference for Batman in January 2018, S_1, is given below.

$$S_1 = \begin{array}{c} \begin{bmatrix} 20\,000 \\ 45\,000 \\ 80\,000 \end{bmatrix} \end{array} \begin{array}{c} I \\ N \\ Y \end{array}$$

b. How many citizens are expected to change their vote to 'Yes' next month?

1 mark

c. The information in the transition diagram has been used to write the transition matrix, T, shown below.

$$\begin{array}{cc} & \textit{this month} \\ & \begin{array}{ccc} I & N & Y \end{array} \\ T = & \begin{bmatrix} 0.85 & 0.05 & 0 \\ 0.05 & 0.90 & 0.05 \\ 0.10 & 0.05 & 0.95 \end{bmatrix} \end{array} \begin{array}{c} \\ \\ I \\ N \\ Y \end{array} \quad \textit{next month}$$

i. Evaluate the matrix $S_4 = T^3 S_1$ and write it down in the space below. 1 mark
Write the elements correct to the nearest whole number.

$$S_4 = \begin{bmatrix} \text{\_\_\_\_\_\_\_\_} \\ \text{\_\_\_\_\_\_\_\_} \\ \text{\_\_\_\_\_\_\_\_} \end{bmatrix}$$

ii. What information does Matrix S_4 contain?

1 mark

Question 3 (4 marks)

A boxing ring sells Champagne (C), Merchandise (M) and Sliders (S). The number of each item sold, over the first three rounds of a particular fight, is shown in the matrix below.

$$\begin{array}{cc} & \begin{array}{ccc} C & M & S \end{array} \\ X = & \begin{bmatrix} 169 & 131 & 174 \\ 159 & 178 & 112 \\ 149 & 95 & 195 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Round 1} \\ \text{Round 2} \\ \text{Round 3} \end{array}$$

a. In total, how many Champagnes are sold over the first three rounds of the fight?

1 mark

The element in Row i and Column j of Matrix X is x_{ij}.

b. What does the element x_{23} indicate?

1 mark

c. Consider the matrix equation

$$\begin{bmatrix} 169 & 131 & 174 \\ 159 & 178 & 112 \\ 149 & 95 & 195 \end{bmatrix} \times \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 5637.05 \\ 5554.20 \\ 5127.35 \end{bmatrix}$$

where $a =$ Cost of one Champagne, $b =$ Cost of one piece of Merchandise, $c =$ Cost of one Slider.

i. What is the cost of one Slider?

1 mark

The matrix equation below shows that the total value of all Champagne, Merchandise and Sliders sold in Round 1 and Round 3 is \$10,764.40.

$$Y \times \begin{bmatrix} 5637.05 \\ 5554.20 \\ 5127.35 \end{bmatrix} = \begin{bmatrix} 10\ 764.40 \end{bmatrix}$$

ii. Matrix Y in this equation is of order 1×3. Write down Matrix Y.

1 mark

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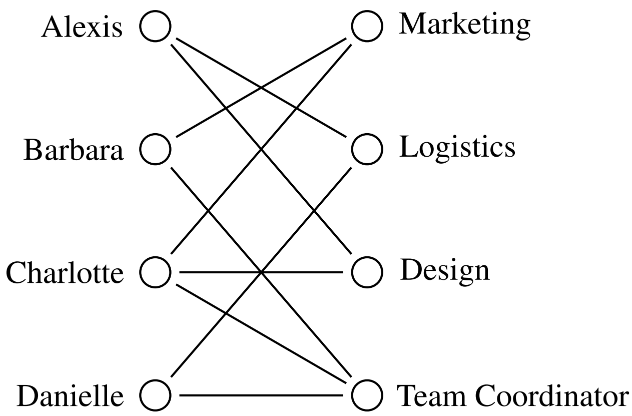
Module 2 - Networks and decision mathematics

Question 1 (2 marks)

Four girls, Alexis, Barbara, Charlotte and Danielle, organise pop-up nightclubs. The edges of the bipartite graph below show the jobs that each of the girls can take on in preparation.



a. How many of the girls are able to take on the 'Team Coordinator' role?

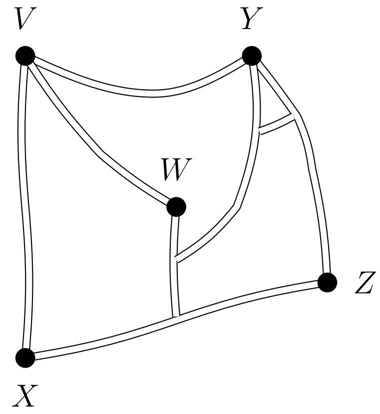
1 mark

b. Which role are both Alexis and Charlotte able to take on?

1 mark

Question 2 (3 marks)

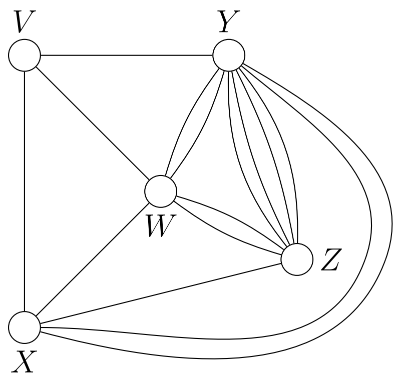
A map of the trails connecting five cabins at a snow resort, V, W, X, Y, Z, is shown below.



a. Starting at V, which cabins can be reached using only one trail?

1 mark

A graph that represents the map of the trails is shown below.



b. **i.** Between which two vertices is an edge missing?

1 mark

ii. Explain why the map shows only two trails leaving W, when the graph shows six edges leaving W.

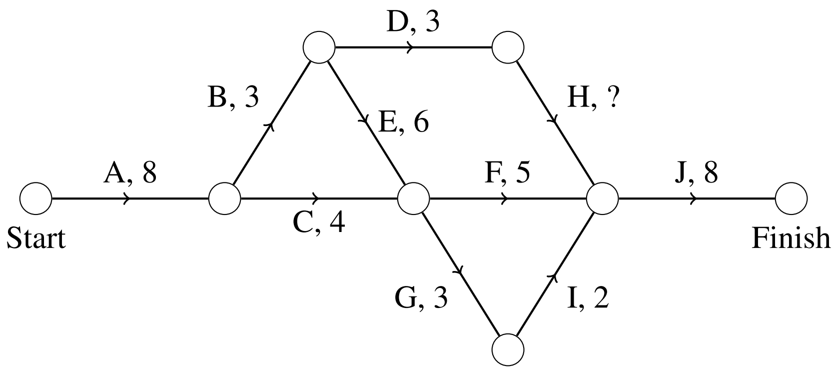
1 mark

Question 3 (7 marks)

Ten activities are needed to prepare the daily shipments of products from Technika to their distributors. The duration in minutes, earliest starting time (EST) and immediate predecessors for these activities are shown in the table below.

Activity	Duration	EST	Predecessor(s)
A	8	0	-
B	3	8	A
C	4	8	A
D	3	11	B
E	6	11	B
F	5	17	C, E
G	3	17	C, E
H		14	D
I	2		G
J	8	23	F, H, I

The directed network that shows these activities is shown below.



All ten of these activities can be completed in a minimum of 31 minutes.

a. What is the EST of activity I?

1 mark

b. What is the latest starting time of activity F?

1 mark

c. Given that the EST of activity J is 23 minutes, what is the duration of activity H?

1 mark

d. Write down, in order, the activities along the critical path.

1 mark

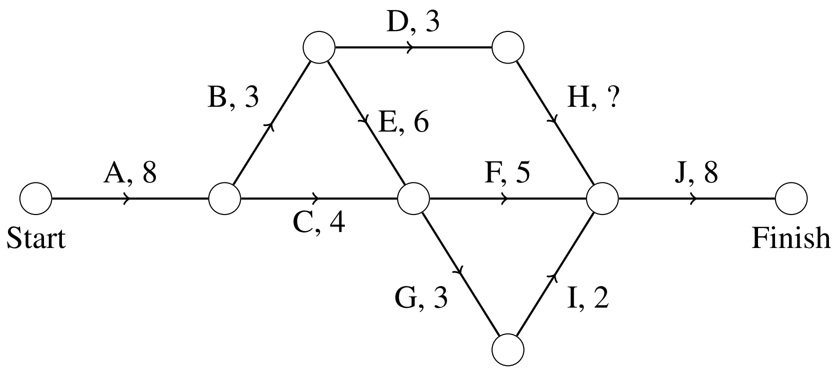
Activities B and C can only be completed by either Max or Joe. One Friday, Max is on holiday and both activities B and C must be completed by Joe. Joe must complete one of these activities before starting the other.

e. What is the minimum effect of this on the usual minimum preparation time for the shipment to the distributors?

1 mark

f. Once a month, Technika attaches promotional material to their shipments. This causes a slight change to activity H, which then cannot start until activities E and C have been completed.

i. On the graph below, show this change without duplicating any activity. 1 mark



ii. What effect does the inclusion of promotional material have on the usual minimum preparation time for shipments to distributors?

1 mark

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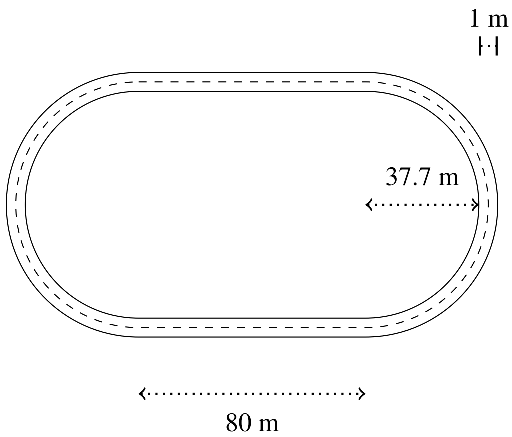
Module 3 - Geometry and measurement

Question 1 (3 marks)

To practice for the Olympics, Brayden runs a lane at a practice track with a straight length of 80 metres, internal radius of 37.7 metres, and a lane width of 1 metre, as shown in the figure below.



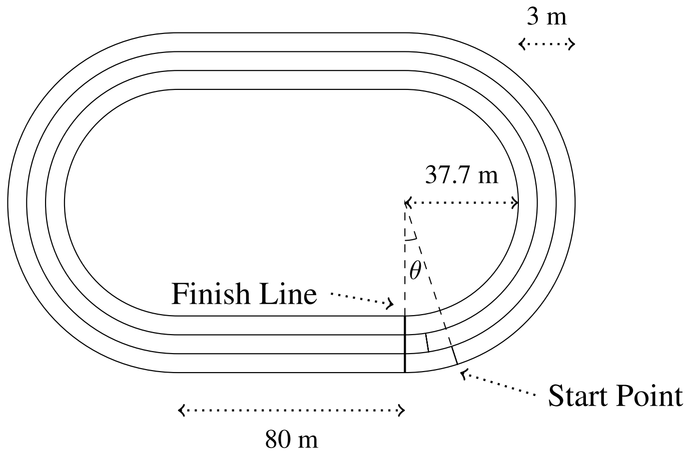
Assume that Brayden runs in the perfect middle of the lane, shown by the dotted path in the above figure.

a. Show that the distance that Brayden runs in a single lap is 400 metres, correct to the nearest metre.

---

1 mark

The entire track of three lanes is shown below. Each extra lane is an additional 1 metre further from the inside lane. The starting positions of each additional lane will be further down the lane to keep the lap distance the same, so that each lane finishes at the same Finish Line.

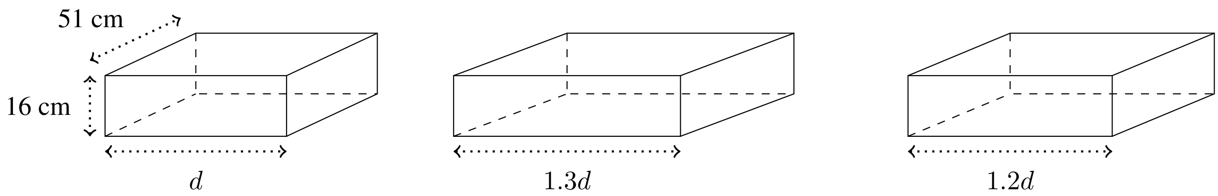


b. If Brayden starts running in the outer lane, as shown by the Start Point, what is the angle θ, in degrees, that is required for Brayden to run the same lap length as the inner lane? Round your answer to the nearest degree.

2 marks

Question 2 (4 marks)

Dylan is cleaning a couch. This couch has three seat cushions in the shapes of rectangular prisms. All three cushions have the same height and depth but different lengths as shown in the figures below.



The cushions have a height of 16 cm.

The cushions have a depth of 51 cm.

The volume of the largest cushion is 74,256 cm^3.

a. What is the value of d, in centimetres? Round your answer to the nearest centimetre.

1 mark

b. What is the total volume, in cubic centimetres, of all three cushions? Round your answer to the nearest cubic centimetre.

1 mark

Dylan realises he is being silly and that the second largest cushion is from a different couch. In a futile attempt to make a beanbag, he tears the second largest cushion apart and tapes it back together in the shape of a sphere.

c. Assuming the volume remains the same, what is the radius, in centimetres, of Dylan's makeshift beanbag? Round your answer to one decimal place.

2 marks

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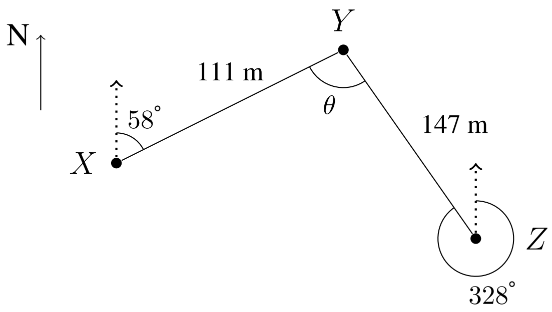
Question 3 (5 marks)

Sam is navigating through a South American jungle. He travels from X to Y, and then from Y to Z. Sam's path is shown in the diagram below.



Sam travelled 111 metres on a bearing of 58° from point X to point Y.

Sam got lost on his way to point Z but knew that point Y was 147 metres away on a bearing of 328° from point Z.

a. What is the bearing, in degrees, of point Z from point Y?

1 mark

b. What is the angle θ, in degrees?

1 mark

Sam will take a straight-line path from point Z back to point X.

c. What is the distance, in metres, that Sam will have to travel to return to point X? Round your answer to the nearest metre.

1 mark

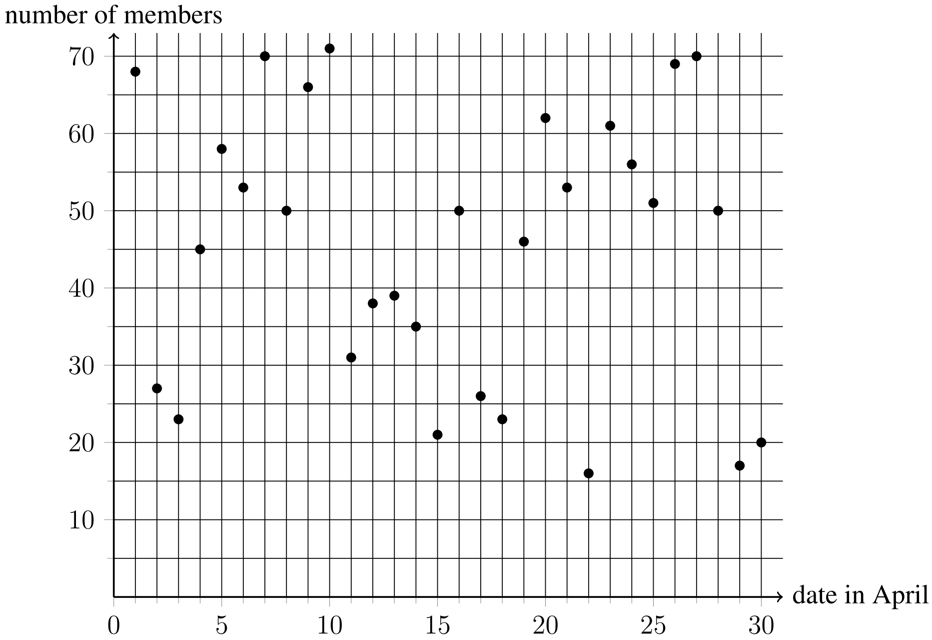
d. What is the bearing, in degrees, of point X from point Z? Round your answer to the nearest degree.

2 marks

Module 4 - Graphs and relations

Question 1 (2 marks)

The graph below shows the number of members that went to Average Joe's Gym each day during April.



a. How many members went to Average Joe's on the 16th of April?

1 mark

b. On how many days in April did more than 65 members go to Average Joe's Gym?

1 mark

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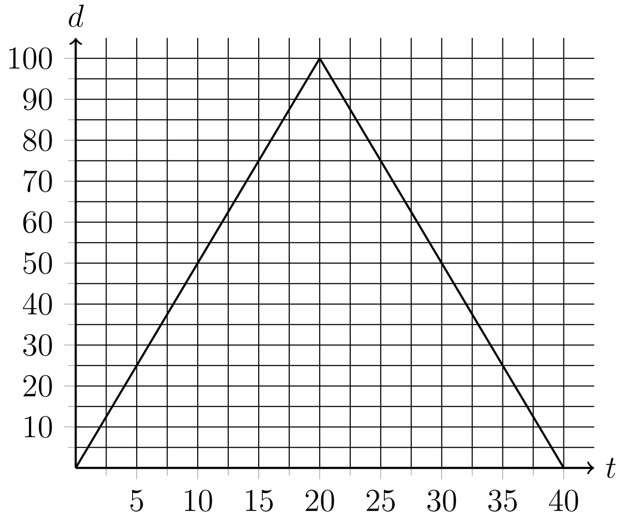
Question 2 (5 marks)

Grade 2 schoolboys Tommy and Jacob have long contested over who the fastest runner is of the two. It is decided that they will race. The first boy to run to the tree 100 metres away and then back to their starting position will be crowned fastest in all the schoolyard.

Tommy reaches the tree after 20 seconds and finishes the race in 40 seconds. The graph below shows his distance d from his starting position, in metres, t seconds after the race begins.



a. What distance will Tommy have run after 25 seconds?

1 mark

Let A be Tommy's distance from his starting position, in metres, t minutes after the race starts. The linear relation that represents his run to the tree is of the form

$$A = kt, \qquad \text{where } 0 \leq t \leq 20$$

The slope of the line, k, is the speed at which Tommy is running in metres per second.

b. Show that $k = 5$.

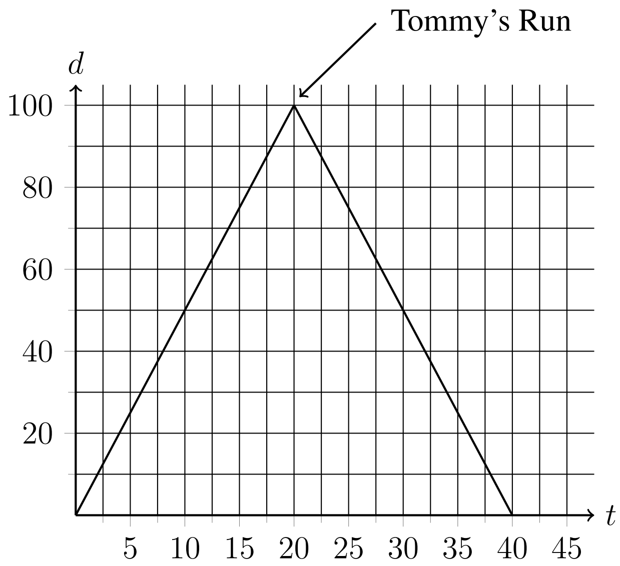
1 mark

Below is the relation that describes Jacob's run, where B is his distance from his starting position, in metres, t seconds after the race begins. M is the time it took Jacob to reach the tree and N is the time it took Jacob to finish the race.

$$B = \begin{cases} \frac{20t}{3}, & 0 \le t \le M \\ -\frac{10t}{3} + 150, & M < t \le N \end{cases}$$

The graph below again shows the relation representing Tommy's run.

c. Sketch the relation representing Jacob's run on the axes below. 2 marks



d. How many seconds after the start of the race did Tommy catch up to Jacob?

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1 mark

Question 3 (5 marks)

Maya makes homemade dog treats and sells them online.

Let x be the number of beef treats Maya makes each week.

Let y be the number of chicken treats Maya makes each week.

The inequalities below represent the constraints on the number of each treat that can be made each week.

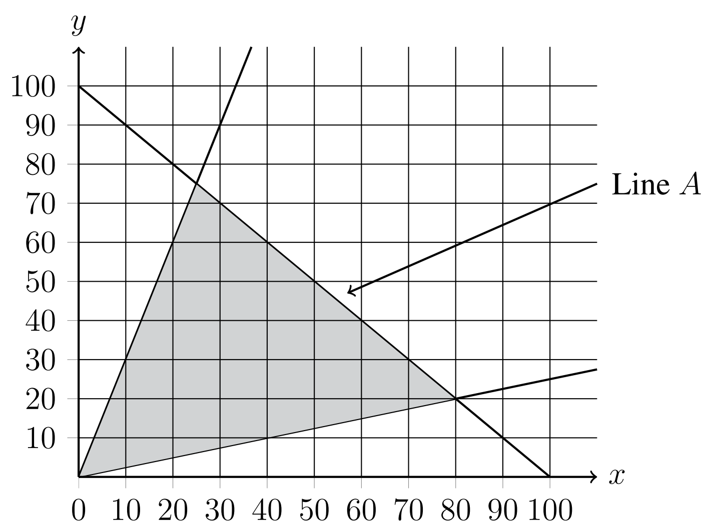
Constraint 1:	$x \geq 0$
Constraint 2:	$y \geq 0$
Constraint 3:	$y \leq 3x$
Constraint 4:	$4y \geq x$

a. Interpret Constraint 3 in terms of the number of beef treats and the number of chicken treats produced each week.

1 mark

There is another constraint, Constraint 5, on the number of each treat that can be made each week.

Constraint 5 is bounded by Line A, shown by the graph below.



The shaded region of the graph contains the points that satisfy Constraints 1 to 5.

b. Write down the inequality that represents Constraint 5.

1 mark

The profit, P, that Maya makes from selling the dog treats is given by

$$P = 3.7x + 4.5y$$

c. Find the maximum profit that Maya can make from the sale of dog treats each week.

1 mark

Maya wants to change the selling price of her beef and chicken dog treats in order to increase her maximum profit to $447.

All of the constraints on the numbers of beef and chicken treats that can be made each week remain the same.

The profit, Q, that is made from the sale of dog treats is now given by

$$Q = ix + jy$$

The profit made on the beef treats is i dollars per treat.

The profit made on the chicken treats is j dollars per treat.

The maximum profit of $447 is made by selling 41 beef treats and 59 chicken treats.

d. What are the values of i and j? Write your answer correct to two decimal places.

2 marks

END OF QUESTION AND ANSWER BOOK

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FURTHER MATHEMATICS

Written examination 2

FORMULA SHEET

Instructions

This formula sheet is provided for your reference. A question and answer book is provided with this formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Formula Sheet

Core - Data Analysis

standardised score	$z = \dfrac{x - \bar{x}}{s_x}$
lower and upper fence in a boxplot	lower $Q_1 - 1,5 \times IQR$ upper $Q_3 + 1.5 \times IQR$
least squares line of best fit	$y = a + bx$, where $b = r\dfrac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$
residual value	residual value $=$ actual value $-$ predicted value
seasonal index	seasonal index $= \dfrac{\text{actual figure}}{\text{deseasonalised figure}}$

Core - Recursion and Financial Modelling

first-order linear recurrence relation	$u_0 = a, \quad u_{n+1} = bu_n + c$
effective rate of interest for a compound interest loan or investment	$r_{\text{effective}} = \left[\left(1 + \dfrac{r}{100n}\right)^n - 1\right] \times 100\%$

Module 1 - Matrices

determinant of a 2×2 matrix	$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$
inverse of a 2×2 matrix	$A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, where $\det A \neq 0$
recurrence relation	$S_0 =$ initial state, $S_{n+1} = TS_n + B$

Module 2 - Networks and Decision Mathematics

Euler's formula	$v + f = e + 2$

Module 3 - Geometry and Measurement

area of a triangle	$A = \frac{1}{2}bc\sin(\theta°)$
Heron's formula	$A = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{1}{2}(a+b+c)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule	$a^2 = b^2 + c^2 - 2bc\cos(A)$
circumference of a circle	$2\pi r$
length of an arc	$r \times \frac{\pi}{180} \times \theta°$
area of a circle	πr^2
area of a sector	$\pi r^2 \times \frac{\theta°}{360}$
volume of a sphere	$\frac{4}{3}\pi r^3$
surface area of a sphere	$4\pi r^2$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a prism	area of base $\times$ height
volume of a pyramid	$\frac{1}{3} \times$ area of base $\times$ height

Module 4 - Graphs and Relations

gradient (slope) of a straight line	$m = \frac{y_2 - y_1}{x_2 - x_1}$
equation of a straight line	$y = mx + c$

END OF FORMULA SHEET