TRIUMPH PUBLICATIONS

Fortify Sample Exam 2B

FURTHER MATHEMATICS

Written examination 2

Reading time: 15 minutes
Writing time: 1 hour 30 minutes

QUESTION AND ANSWER BOOK

Structure of book

Section A – Core	*Number of questions*	*Number of questions to be answered*	*Number of marks*
	6	6	36
Section B – Modules	*Number of modules*	*Number of modules to be answered*	*Number of marks*
	4	2	24
			Total 60

- Students are to write in blue or black pen.
- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, one bound reference, one approved technology (calculator or software) and, if desired, one scientific calculator. Calculator memory DOES NOT need to be cleared. For approved computer-based CAS, full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer book of 35 pages.
- Formula sheet
- Working space is provided throughout the book.

Instructions
- Write your **name** and **student number** in the space provided above on this page.
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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SECTION A - Core

Instructions for Section A

Answer **all** questions in the spaces provided.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.

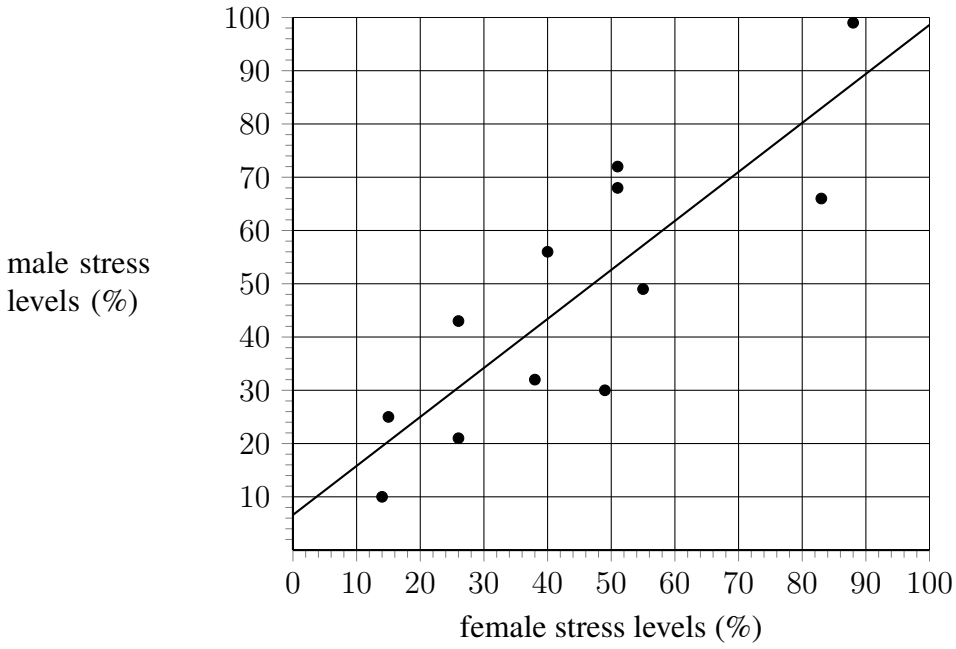
In 'Recursion and financial modelling', all answers should be rounded to the nearest cent unless otherwise instructed.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Data Analysis

Question 1 (3 marks)

The scatterplot below plots male stress levels (*male*) against female stress levels (*female*) in year 12 for a number of schools. A least squares regression line has been fitted to the scatterplot as shown.



The slope of this least squares regression line is 0.92.

a. Interpret the slope in terms of the variables *male* stress levels and *female* stress levels.

1 mark

The equation of this least squares regression line is

$$male = 6.6 + 0.92 \times female$$

b. In a particular school, *female* stress levels were at 40%.
Use the equation to predict *male* stress levels for that school.

---

1 mark

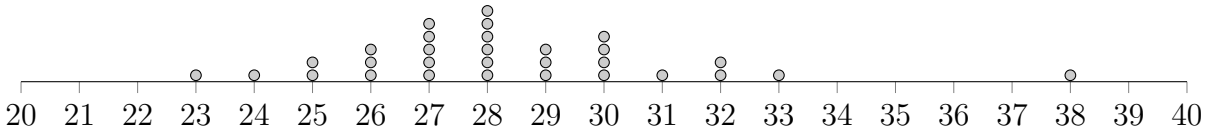
c. The coefficient of determination is 0.71. Interpret the coefficient of determination in terms of male stress levels and female stress levels.

---

1 mark

Question 2 (3 marks)

The dot plot below displays the number of times a student curses each day in the 30 days leading up to their exams.



a. For this plot determine

i. the median number of times this student cursed

1 mark

ii. the percentage of days on which the student cursed 30 times or more.

1 mark

The number of times this student cursed is approximately normally distributed with a mean of 28.4 and a standard deviation of 3.0.

b. Determine the percentage of days out of the 30 that the student is expected to curse less than 22.4 times.

Write your answer correct to one decimal place.

1 mark

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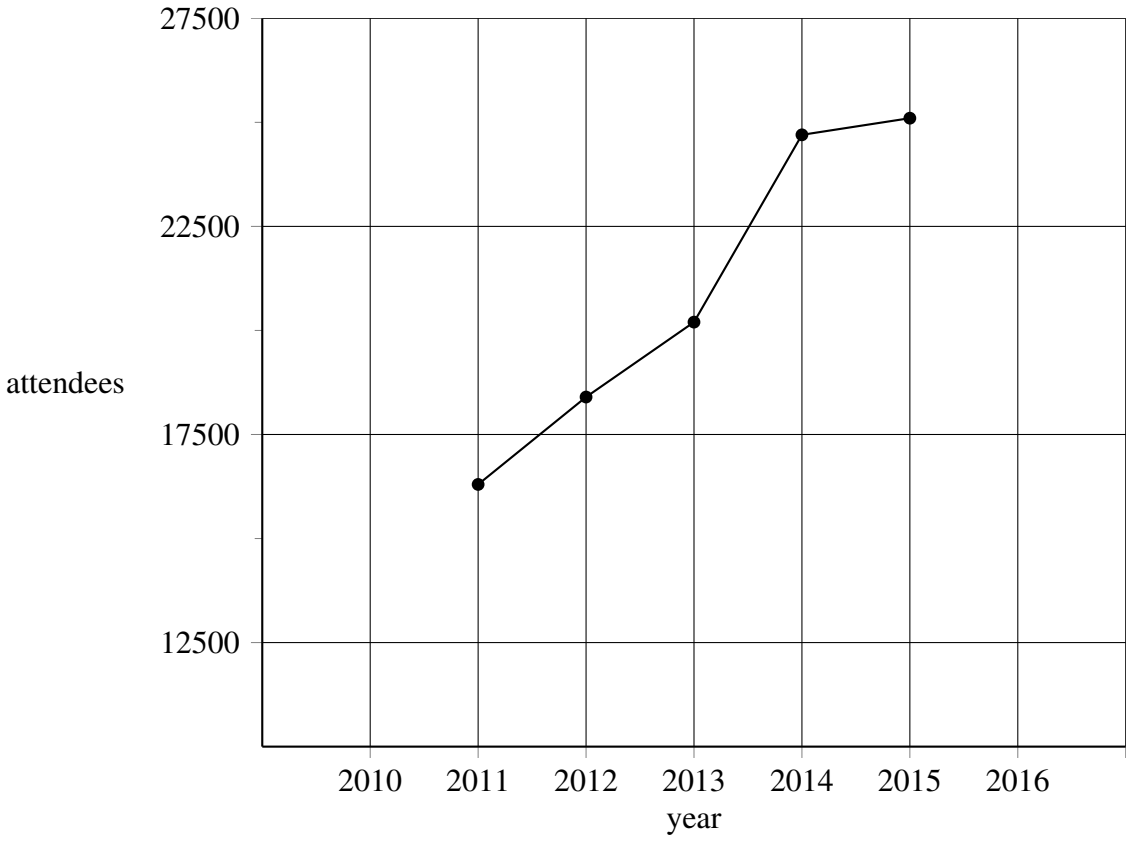
Question 3 (6 marks)

The table below shows the number of attendees at an Australian music festival over yearly intervals for the period 2010 to 2016.

Year	2010	2011	2012	2013	2014	2015	2016
Attendees	14,700	16,300	18,400	20,200	24,700	25,100	24,900



a. Complete the time series plot above by plotting the number of attendees for the years 2010 and 2016. 1 mark

b. Briefly describe the general trend in the data.

_____

1 mark

In the table below, the variable *year* has been rescaled using $2010 = 0$, $2011 = 1$ and so on. The new variable is *time*.

Year	2010	2011	2012	2013	2014	2015	2016
Time	0	1	2	3	4	5	6
Attendees	14,700	16,300	18,400	20,200	24,700	25,100	24,900

c. Use the variables *time* and *attendees* to write down the equation of the least squares regression line that can be used to predict *attendees* from *time*. Take *time* as the explanatory variable.

Write your answer correct to the nearest whole numbers.

---

2 marks

d. In the year 2017, 26,300 people attended the music festival. Calculate the residual value for 2017 if the least squares regression line calculated in **part c.** is used to predict the number of attendees in 2017.

---

2 marks

Recursion and financial modelling

Question 4 (3 marks)

Bernadette invests in an annuity. This annuity investment earns interest at the rate of 7.2% per annum, compounding monthly. Bernadette invests $90,000 and makes withdrawals of $1,500 each month.

a. Write a recurrence relation that describes Bernadette's annuity investment in terms of the value of the investment, V_n, after n months.

2 marks

b. Would changing the number of compounding periods per year from 12 to 1 result in Bernadette receiving more money from her investment or less?

1 mark

Question 5 (4 marks)

Sam has borrowed \$45,000 to build a drone-supported hammock. He will be charged interest at the rate of 5.8% per annum, compounding monthly.

a. For the first two years (24 months), Sam will make monthly repayments of \$550.

i. Find the amount that Sam will owe on his loan after he has made 24 repayments.

1 mark

ii. What is the total interest Sam will have paid after 24 repayments?

1 mark

After four years, Sam will make a lump sum payment of $\$P$ in order to reduce the balance of his loan. The lump sum payment will ensure that Sam's loan is fully repaid in a further two years. Sam's repayment amount remains at \$550 per month and the interest rate remains at 5.8% per annum, compounding monthly.

b. What is the value of Sam's lump sum payment, $\$P$? Give your answer correct to the nearest dollar.

2 marks

Question 6 (5 marks)

Dylan has opened a savings account to save money to buy his first car. The amount of money in the savings account after n years, V_n, can be modelled by the recurrence relation shown below.

$$V_0 = 10\ 000, \qquad V_{n+1} = 1.05 \times V_n$$

a. How much money did Dylan initially deposit into the savings account?

1 mark

b. Use recursion to write down calculations that show that the amount of money in Dylan's savings account after three years, V_3, will be \$11,576.25.

1 mark

c. What is the annual compound interest rate for this savings account?

1 mark

d. The amount of money in the account after n years, V_n, can also be determined using a rule.

i. Complete the rule below by writing the appropriate numbers in the boxes provided.

$$V_n = \square^{n} \times \square$$

1 mark

ii. How much money will be in Dylan's account after 7 years?

1 mark

END OF SECTION A
TURN OVER

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SECTION B - Modules

Instructions for Section B

Select **two** modules and answer **all** questions within the selected modules.

You need not give numerical answers as decimals unless instructed to do so. Alternative forms may include, for example, π, surds or fractions.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Contents **Page**

Module 1 - Matrices

Question 1 (2 marks)

A group of five friends, Lester (L), Marty (M), Nicola (N), Oscar (O) and Penelope (P) are playing a game of Chinese whispers, where each player can only whisper to other players according to matrix C below.

The matrix C^2 is also shown below.

$$C = whisperer \quad \begin{array}{c} \\ L \\ M \\ N \\ O \\ P \end{array} \begin{array}{c} \begin{array}{ccccc} \multicolumn{5}{c}{receiver} \\ L & M & N & O & P \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

$$C^2 = whisperer \quad \begin{array}{c} \\ L \\ M \\ N \\ O \\ P \end{array} \begin{array}{c} \begin{array}{ccccc} \multicolumn{5}{c}{receiver} \\ L & M & N & O & P \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 1 & 2 \\ 1 & 2 & 2 & 0 & 1 \\ 1 & 1 & 3 & 1 & 3 \\ 1 & 0 & 1 & 1 & 2 \end{bmatrix} \end{array}$$

A '1' in matrix C indicates that a whisperer can whisper directly to a receiver.

A '0' in matrix C indicates that a whisperer can not whisper directly to a receiver.

a. To whom can Marty directly whisper?

1 mark

Penelope wants a whisper to reach Oscar but cannot do this directly.

b. Write down the name(s) of the friend(s) who could be whispered to and immediately pass the whisper to Oscar.

1 mark

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Question 2 (4 marks)

Mice, in an experiment around enjoyable activities, are recorded each hour. The mice are able to choose between four activities: Playing with balls (B), napping (N), using the slides (S), and running on the big wheels (W).

The transition matrix T shows the way in which the mice are expected to change their choice of activity from hour to hour.

$$T = \begin{array}{c} \\ B \\ N \\ S \\ W \end{array} \begin{array}{c} \begin{array}{cccc} \textit{this hour} & & & \\ B & N & S & W \end{array} \\ \begin{bmatrix} 0.5 & 0.2 & 0.1 & 0.2 \\ 0.2 & 0.4 & 0.2 & 0.2 \\ 0.2 & 0.1 & 0.6 & 0.2 \\ 0.1 & 0.3 & 0.1 & 0.4 \end{bmatrix} \end{array} \begin{array}{l} B \\ N \\ S \\ W \end{array} \quad \textit{next hour}$$

Let S_n be the state matrix for the number of mice expected to choose each activity at the beginning of each hour n. For the given matrix S_1, a matrix rule that can be used to predict the number of mice at each activity, each hour after beginning is

$$S_1 = \begin{bmatrix} 100 \\ 100 \\ 100 \\ 100 \end{bmatrix}, \qquad S_{n+1} = TS_n$$

a. How many mice will be on the same activity they started on, one hour after beginning?

1 mark

b. Complete S_2, the state matrix for the beginning of hour 2, below. 1 mark

$$S_2 = \begin{bmatrix} \_\_\_\_\_ \\ \_\_\_\_\_ \\ \_\_\_\_\_ \\ \_\_\_\_\_ \end{bmatrix} \begin{matrix} B \\ N \\ S \\ W \end{matrix}$$

c. Of the mice expected to be napping at the beginning of hour 3, what percentage were on the slides at the beginning of hour 2?

2 marks

Question 3 (6 marks)

A small city is divided into four Quadrants: Alpha (A), Beta (B), Charlie (C) and Delta (D). The number of Jans (J), the number of Michaels (M) and the number of Vincents (V) living in each of the quadrants in 2018 is shown in Matrix R below.

$$R = \begin{array}{c} \begin{matrix} J & M & V \end{matrix} \\ \begin{bmatrix} 390 & 450 & 470 \\ 500 & 360 & 410 \\ 550 & 500 & 360 \\ 380 & 460 & 440 \end{bmatrix} \end{array} \begin{matrix} A \\ B \\ C \\ D \end{matrix}$$

a. Write down the order of Matrix R.

---

1 mark

b. How many Michaels lived in Quadrant Charlie in 2018?

---

1 mark

c. In terms of Quadrant Alpha, what does the sum of elements in the first row of Matrix R represent?

---

1 mark

A survey is to be held in the city. All of the Jans, Michaels and Vincents will be asked if they are happy with their name, or would prefer another name.

It is believed that 35% of Jans will be happy with their names, 15% of Michaels will be happy with their names and 50% of Vincents will be happy with their names.

This information is shown in Matrix M below.

$$M = \begin{bmatrix} 0.35 \\ 0.15 \\ 0.50 \end{bmatrix}$$

d. Explain, in terms of rows and columns, why the matrix product $R \times M$ is defined.

1 mark

The product of matrices R and M is shown below.

$$R \times M = \begin{bmatrix} 390 & 450 & 470 \\ 500 & 360 & 410 \\ 550 & 500 & 360 \\ 380 & 460 & 440 \end{bmatrix} \times \begin{bmatrix} 0.35 \\ 0.15 \\ 0.50 \end{bmatrix} = \begin{bmatrix} x \\ 434 \\ 447.5 \\ 422 \end{bmatrix}$$

e. Using appropriate elements from the matrix product $R \times M$, write a calculation to show that the value of x is 439.

1 mark

f. In this small city, how many Jans, Michaels and Vincents are happy with their names? Give your answer correct to the nearest whole person.

1 mark

Module 2 - Networks and decision mathematics

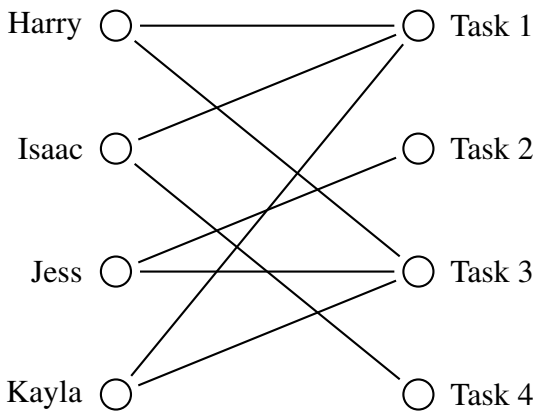
Question 1 (1 mark)

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

The adjacency matrix above represents a planar graph with four vertices.
In the box below, write the number of faces this planar graph has.

1 mark

Question 2 (2 marks) The bipartite graph below shows the tasks that each of four people is able to undertake.



All tasks must be allocated and each person can be allocated one task only.
Write the two possible allocations in the tables below. 2 marks

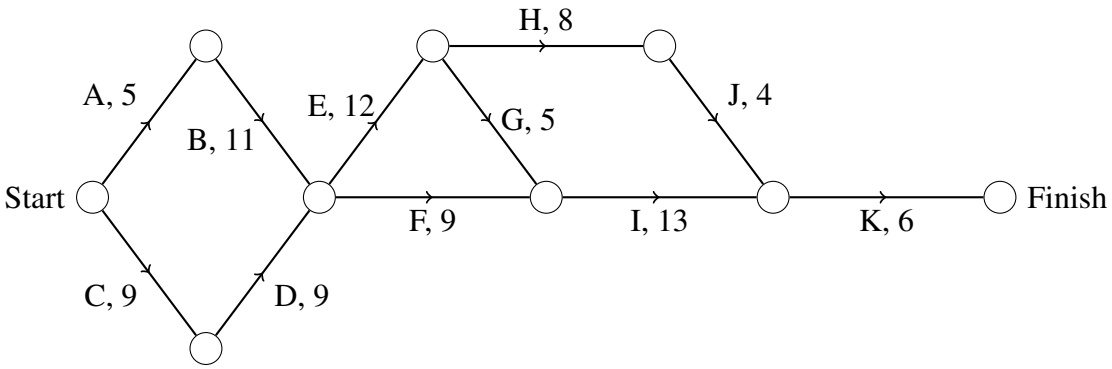
	Task 1
	Task 2
	Task 3
	Task 4

	Task 1
	Task 2
	Task 3
	Task 4

CONTINUES OVER PAGE

Question 3 (5 marks)

To prepare for a corporate function, 11 activities need to be completed. The network below shows these 11 activities and their completion time in hours.



a. Determine the earliest starting time of activity G.

---

1 mark

The minimum completion time in which all 11 activities can be completed is 54 hours.

b. What is the latest starting time of activity J?

---

1 mark

c. What is the float time of activity F?

---

1 mark

Just before they start preparing for the function, the committee members realise the need to add another activity, Q, to the project. Activity Q will take twelve hours to complete. Activity H is the only predecessor of activity Q, and activity Q finishes with the project.

d. What is the latest starting time of activity Q if it is not to increase the minimum completion time of the project?

1 mark

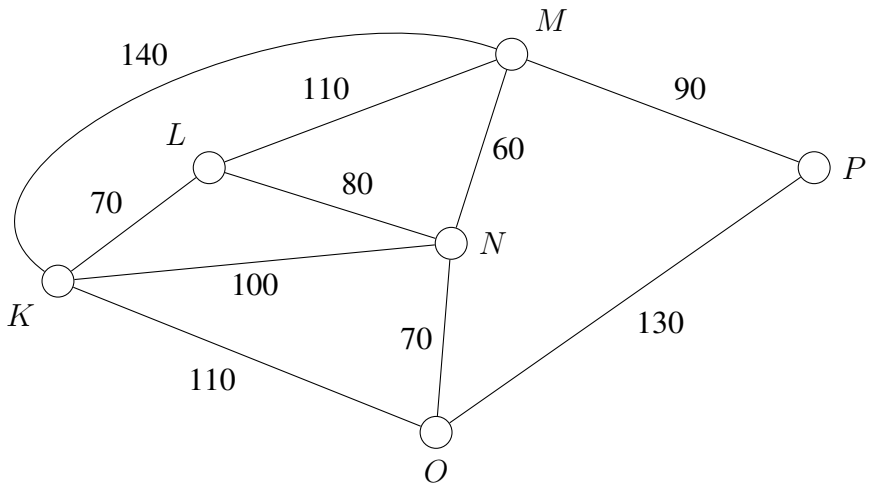
Activity D can be crashed by up to four hours at an additional cost of \$110 per hour. This may reduce the minimum completion time for the project, including activity Q.

e. Determine the least cost of crashing activity D to give the greatest reduction in the minimum completion time of the project.

1 mark

Question 4 (4 marks)

The diagram below shows a network of tunnels between six bases at a paintball arena: K, L, M, N, O and P. The numbers indicate the distances, in metres, that are travelled by tunnel to connected bases.



a. Rubi follows an Eulerian trail through this network of tunnels.

i. Write down the names of the bases at the start and end of Rubi's trail.

_____

_____

1 mark

ii. What distance did she travel?

_____

_____

1 mark

b. Maddie will follow a Hamiltonian path from base P to base L.
What is the shortest distance she can travel?

1 mark

The tunnel between base O and base N is closed off. If one other tunnel is closed off from the network, Coco will be able to follow an Eulerian circuit through the network of tunnels.

c. Which other tunnel should be removed? In the boxes below, write down the pair of bases that this tunnel connects.

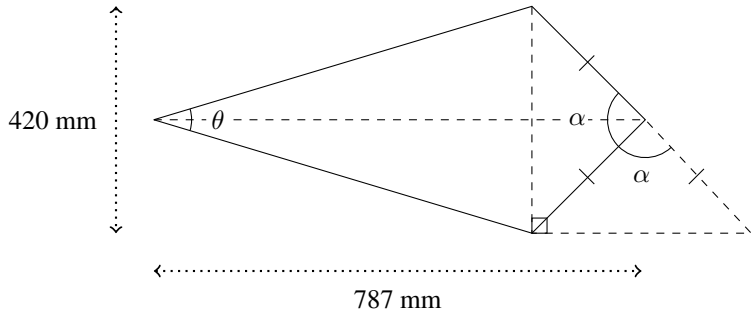
Between ☐ and ☐

1 mark

Module 3 - Geometry and measurement

Question 1 (3 marks)

Little Johnny is designing a kite. His design is shown in the figure below.



The kite has a length of 787 mm.

The kite has a width of 420 mm.

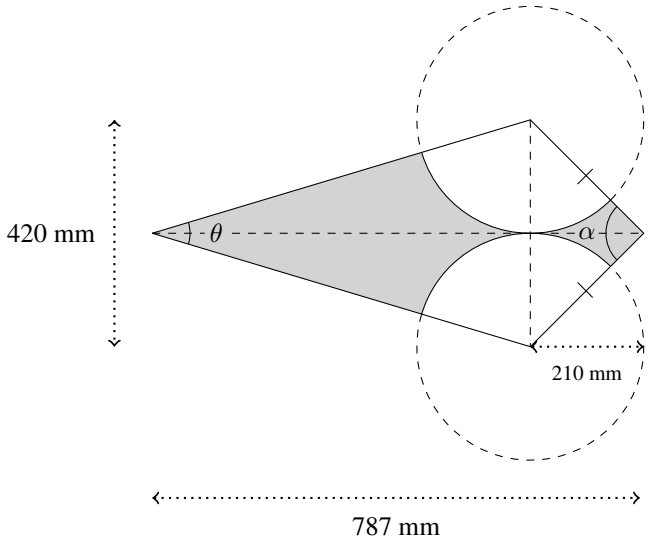
a. **i.** What is the value of the angle α, correct to the nearest degree?

1 mark

ii. What is the value of the angle θ, correct to the nearest degree?

1 mark

Little Johnny, being an artist, is colouring in his kite with circular shapes; the design of the kite is shown in the figure below.



The circles have a radius of 210 mm.

c. What is the area, in square centimetres, of the shaded region? Round your answer to two decimal places.

1 mark

Question 2 (5 marks)

Gordan will travel from Sydney to Los Angeles on Tuesday, 6th March. The flight will leave Sydney at 11:40am, and will take 14 hours and 20 minutes to reach Los Angeles.

At this time of year, Los Angeles is 19 hours behind Sydney time.

a. On what day and at what time will Gordan arrive in Los Angeles?

1 mark

b. Gordan travels East to Atlanta and goes to two nightclubs. The second nightclub is located 112 metres away on a straight-line bearing of 120° from the first nightclub.

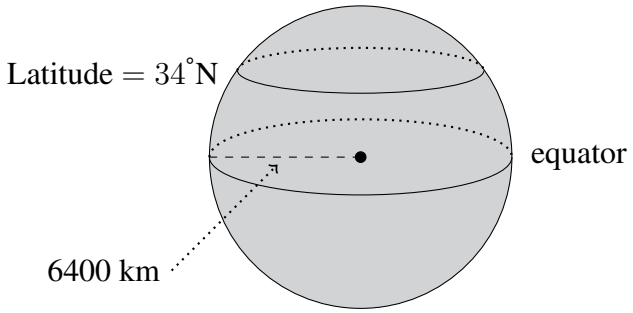
i. What is the eastern distance of the second nightclub from the first nightclub? Round your answer correct to the nearest metre.

1 mark

ii. What is the southern distance between the two nightclubs? Round your answer correct to the nearest metre.

1 mark

In the following questions, assume that the radius of the Earth is 6,400 km.
The line of latitude around the Earth at latitude 34°N is shown in the diagram below.



c. Show that the radius of the line of latitude around the Earth at 34°N is 5306 km, when rounded to the nearest kilometre.

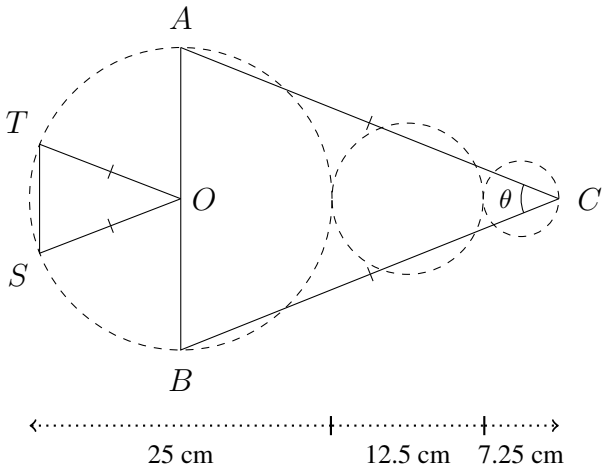
1 mark

d. What is the shortest curved distance between Los Angeles (34°N, 118°W) and Atlanta (34°N, 84°W)? Round your answer to the nearest kilometre.

1 mark

Question 3 (4 marks)

Consider the following diagram.



- The largest circle has a diameter of 25 cm
- The second largest circle has a diameter of 12.5 cm
- The smallest circle has a diameter of 7.25 cm

a. Show that the length, in centimetres, of line BC is 34.59 centimetres when rounded to two decimal places.

1 mark

b. Use the cosine rule to show that the angle θ, correct to one decimal place, is equal to 42.4°.

1 mark

Triangles ABC and TSO are similar.

c. What is the length, in centimetres, of line ST? Round your answer to the nearest centimetre.

1 mark

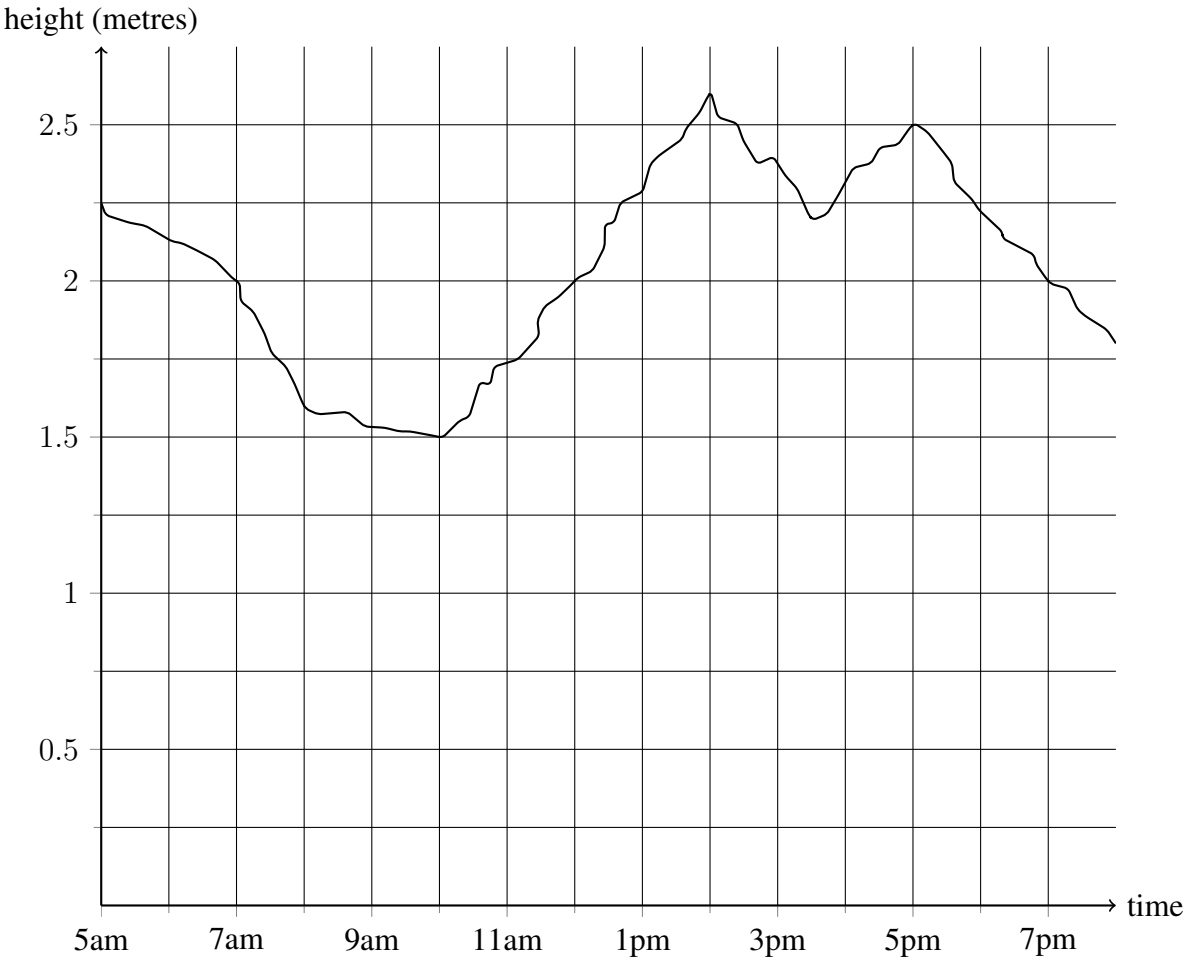
d. What is the area, in square centimetres, of triangle TSO? Round your answer to the nearest square centimetre.

1 mark

Module 4 - Graphs and relations

Question 1 (2 marks)

The graph below shows the average height of waves at a particular beach between 5:00am and 8:00pm.



a. At what time was the average wave height at its lowest?

_____

1 mark

b. For how many hours, between 5:00am and 7:00pm, was the the average wave height greater than 2.0 metres?

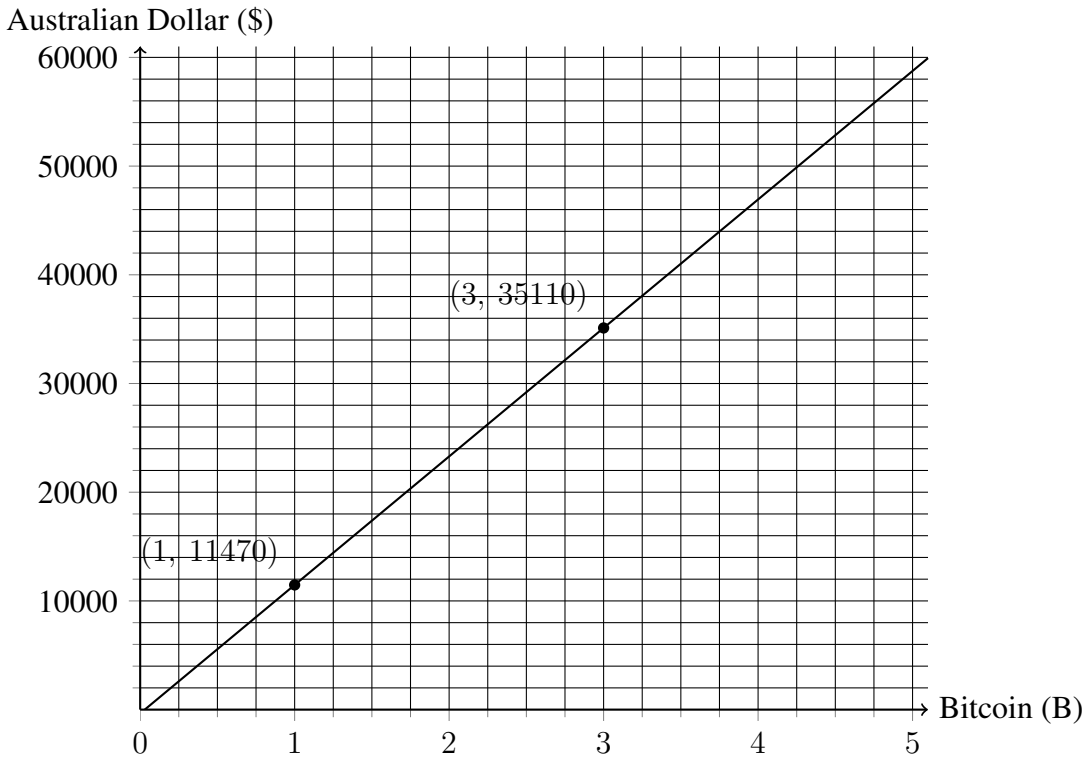
_____

_____

1 mark

Question 2 (2 marks)

The graph below shows the relationship between the Bitcoin and the Australian Dollar on a particular day.



The points $(1, 11470)$ and $(3, 35110)$ are labelled.

The equation for the relationship between the **Australian Dollar** and the **Bitcoin** is

$$\text{Australian Dollar} = 11820 \times \text{Bitcoin} - k$$

a. Use the point $(3, 35110)$ to show that the value of k is 350.

1 mark

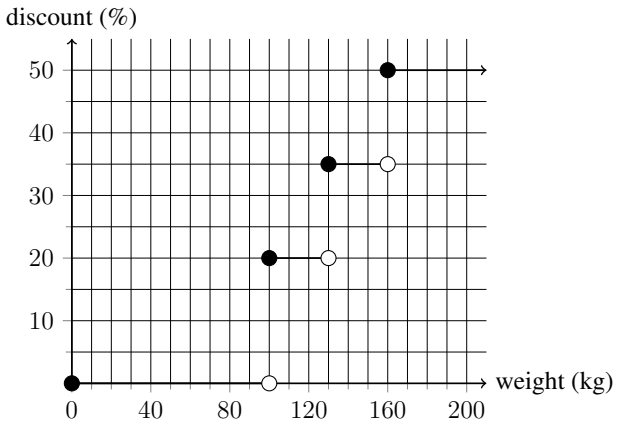
b. Interpret the y-intercept in the context of converting Bitcoin to Australian Dollars.

1 mark

Question 3 (5 marks)

A restaurant offers loyalty bonuses to its heavier, more valuable customers in the form of a discount. This discount is based on the customer's weight, in kilograms.

The graph below can be used to calculate the discount, as a percentage, a customer is to receive based on their weight, in kilograms.



a. Find the discount, as a percentage, that a customer would receive if they weighed 130 kilograms.

1 mark

A competing restaurant catches wind of the weight discount and starts their own. The equation for the relationship between discount, as a percentage, and weight, in kilograms, is

$$\textit{Competing discount} = \begin{cases} \dfrac{\textit{weight}}{5} + 5, & \textit{weight} < 175 \\ 55, & 175 \leq \textit{weight} \end{cases}$$

b. Find the competing restaurant's discount, as a percentage, for a customer that weighs 80 kilograms.

1 mark

Question 4 (3 marks)

Jeremiah sells succulents at his local Farmers Market. He buys the succulents for $2.80 each.

Jeremiah offers a discount to his customers as they buy more succulents at once.

The revenue, in dollars, Jeremiah receives from one customer is given by

$$\textit{Revenue} = \begin{cases} 4.2s, & 0 < s \leq 5 \\ a + 3(s-5), & 5 < s \leq 20 \\ 66 + 2(s-20), & 20 < s \end{cases}$$

where s is the number of succulents that a customer buys at once. The revenue function is continuous.

a. Show that a has a value of 21 in the revenue equation above.

1 mark

b. What is the revenue Jeremiah receives for selling 15 succulents to one customer?

1 mark

c. What is the maximum number of succulents Jeremiah can sell to one customer before making a loss?

1 mark

END OF QUESTION AND ANSWER BOOK

TRIUMPH
PUBLICATIONS

FURTHER MATHEMATICS

Written examination 2

FORMULA SHEET

Instructions

This formula sheet is provided for your reference.
A question and answer book is provided with this formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

Formula Sheet

Core - Data Analysis

standardised score	$z = \dfrac{x - \bar{x}}{s_x}$
lower and upper fence in a boxplot	lower $Q_1 - 1,5 \times IQR$ upper $Q_3 + 1.5 \times IQR$
least squares line of best fit	$y = a + bx$, where $b = r\dfrac{s_y}{s_x}$ and $a = \bar{y} - b\bar{x}$
residual value	residual value = actual value − predicted value
seasonal index	seasonal index $= \dfrac{\text{actual figure}}{\text{deseasonalised figure}}$

Core - Recursion and Financial Modelling

first-order linear recurrence relation	$u_0 = a, \quad u_{n+1} = bu_n + c$
effective rate of interest for a compound interest loan or investment	$r_{\text{effective}} = \left[\left(1 + \dfrac{r}{100n}\right)^n - 1\right] \times 100\%$

Module 1 - Matrices

determinant of a 2×2 matrix	$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $\det A = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$
inverse of a 2×2 matrix	$A^{-1} = \dfrac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, where $\det A \neq 0$
recurrence relation	$S_0 =$ initial state, $S_{n+1} = TS_n + B$

Module 2 - Networks and Decision Mathematics

Euler's formula	$v + f = e + 2$

Module 3 - Geometry and Measurement

area of a triangle	$A = \frac{1}{2}bc\sin(\theta^\circ)$
Heron's formula	$A = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{1}{2}(a+b+c)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule	$a^2 = b^2 + c^2 - 2bc\cos(A)$
circumference of a circle	$2\pi r$
length of an arc	$r \times \frac{\pi}{180} \times \theta^\circ$
area of a circle	πr^2
area of a sector	$\pi r^2 \times \frac{\theta^\circ}{360}$
volume of a sphere	$\frac{4}{3}\pi r^3$
surface area of a sphere	$4\pi r^2$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a prism	area of base $\times$ height
volume of a pyramid	$\frac{1}{3} \times$ area of base $\times$ height

Module 4 - Graphs and Relations

gradient (slope) of a straight line	$m = \frac{y_2 - y_1}{x_2 - x_1}$
equation of a straight line	$y = mx + c$

END OF FORMULA SHEET