TRIUMPH
PUBLICATIONS

Fortify Sample Exam A1

SPECIALIST MATHEMATICS

Written examination 1

Reading time: 15 minutes
Writing time: 1 hour

QUESTION AND ANSWER BOOK

Structure of book

Number of questions	*Number of questions to be answered*	*Number of marks*
10	10	40

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners and rulers.
- Students are NOT permitted to bring into the examination room: any technology (calculators or software), notes of any kind, blank sheets of paper and/or correction fluid/tape.

Materials supplied
- Question and answer booklet of 7 pages.
- Formula sheet.
- Working space is provided throughout the book.

Instructions
- Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.
- All written responses must be in English.

At the end of the examination
- You may keep the formula sheet.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

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Instructions

Answer **all** questions in the spaces provided.

Unless otherwise specified, an **exact** answer is required to a question.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Take the **acceleration due to gravity** to have magnitude g ms^{-2}, where $g = 9.8$.

Question 1 (5 marks)

a. Simplify $\dfrac{\sqrt{3}-i}{(\sqrt{2}+\sqrt{2}i)^5}$, and express the answer in the form of $a\text{cis}(b)$, where a and b are real numbers. 3 marks

b. Solve the equation $z^2 + 2z + 5 = 0$ over $z \in C$. 2 marks

Question 2 (2 marks)

A sample of 49 batteries was found to last an average of 15 hours. If the standard deviation of batter life is known to be 20 minutes, then find the 95% confidence interval for the mean time that the battery will last. Give your answer in the form $a \pm b$ where a and b are real constants.

Question 3 (4 marks)

Find the equation of the normal to the curve given by $\sin(y) = \cos^2(x)$ at the point $\left(\frac{\pi}{4}, \frac{\pi}{6}\right)$.

Question 4 (3 marks)

A particle moves under a force $\underset{\sim}{\mathrm{F}}$ so that its position vector $\underset{\sim}{\mathrm{r}}$ at any time t is given by $\underset{\sim}{\mathrm{r}} = \cos(t)\underset{\sim}{\mathrm{i}} + t^2\underset{\sim}{\mathrm{j}} + \mathrm{e}^{2t}\underset{\sim}{\mathrm{k}}$. Distances are measured in metres and time is measured in seconds.

Calculate the magnitude of the acceleration at π seconds.

Question 5 (4 marks)

Find the volume of the solid of revolution when the region bound by the curve $y = \log_e(3 - x)$, the x-axis and the y-axis is rotated about the y-axis.

Question 6 (4 marks)

Consider the vectors $\underset{\sim}{a} = 2\underset{\sim}{i} - 2\underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{b} = 2\underset{\sim}{i} - \underset{\sim}{j} + 3\underset{\sim}{k}$. Find the vector resolute of b in the direction of a.

Question 7 (4 marks)

The rate of radioactive decay for a particular substance is proportional to the amount Q of the substance at any time t. The differential equation for this situation is $\frac{dQ}{dt} = -kQ$, where k is a constant.

Given that $Q = 20$ when $t = 0$ and $Q = 10$ when $t = 5$, find an equation for Q in terms of t.

Question 8 (4 marks)

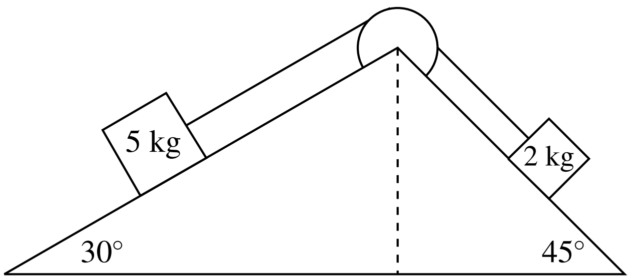
Find the area under the curve $y = \frac{\mathrm{e}^x}{\mathrm{e}^{2x} - 2\mathrm{e}^x + 1}$ from $x = 1$ to $x = 2$. Express your answer as a single fraction.

Question 9 (4 marks)

Let $f(x) = \dfrac{1}{\arccos(x)}$. Find $f'(x)$ and define the domain of which $f'(x)$ is defined.

Question 10 (6 marks)

Two masses of 5 kg and 2 kg are placed on a smooth inclined plane of 30° and 45°.



a. Show all forces acting on both masses on the diagram. 2 marks

b. Find the acceleration of the 5 kg mass down the plan in terms of g. 2 marks

c. Find the tension in the rope in terms of g. 2 marks

END OF QUESTION AND ANSWER BOOK

SPECIALIST MATHEMATICS

Written examination 1

FORMULA SHEET

Instructions

This formula sheet is provided for your reference.
A question and answer book is provided with this formula sheet.

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Formula Sheet

Mensuration

area of a trapezium	$\frac{1}{2}(a+b)h$
curved surface area of a cylinder	$2\pi rh$
volume of a cylinder	$\pi r^2 h$
volume of a cone	$\frac{1}{3}\pi r^2 h$
volume of a pyramid	$\frac{1}{3}Ah$
volume of a sphere	$\frac{4}{3}\pi r^3$
area of a triangle	$\frac{1}{2}bc\sin(A)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
cosine rule	$c^2 = a^2 + b^2 - 2ab\cos(C)$

Circular functions

$\cos^2(x) + \sin^2(x) = 1$	
$1 + \tan^2(x) = \sec^2(x)$	$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$
$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$	$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$
$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$	$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
$\tan(x+y) = \frac{\tan(x)+\tan(y)}{1-\tan(x)\tan(y)}$	$\tan(x-y) = \frac{\tan(x)-\tan(y)}{1+\tan(x)\tan(y)}$
$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$	
$\sin(2x) = 2\sin(x)\cos(x)$	$\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$

Circular functions – continued

Function	$\sin^{-1}$ or arcsin	$\cos^{-1}$ or arccos	$\tan^{-1}$ or arctan
Domain	$[-1,\ 1]$	$[-1,\ 1]$	R
Range	$\left[-\frac{\pi}{2},\ \frac{\pi}{2}\right]$	$[0,\ \pi]$	$\left(-\frac{\pi}{2},\ \frac{\pi}{2}\right)$

Algebra (complex numbers)

$z = x + iy = r\left(\cos(\theta) + i\sin(\theta)\right) = r\mathrm{cis}(\theta)$	
$\lvert z\rvert = \sqrt{x^2 + y^2} = r$	$-\pi < \mathrm{Arg}(z) \leq \pi$
$z_1 z_2 = r_1 r_2 \mathrm{cis}(\theta_1 + \theta_2)$	$\frac{z_1}{z_2} = \frac{r_1}{r_2}\mathrm{cis}(\theta_1 - \theta_2)$
$z^n = r^n \mathrm{cis}(n\theta)$ (de Moivre's theorem)	

Probability and statistics

for random variables X and Y	$\mathrm{E}(aX + b) = a\mathrm{E}(X) + b$ $\mathrm{E}(aX + bY) = a\mathrm{E}(X) + b\mathrm{E}(Y)$ $\mathrm{var}(aX + b) = a^2\mathrm{var}(X)$
for independent random variables X and Y	$\mathrm{var}(aX + bY) = a^2\mathrm{var}(X) + b^2\mathrm{var}(Y)$
approximate confidence interval for μ	$\left(\bar{x} - z\frac{s}{\sqrt{n}},\ \bar{x} + z\frac{s}{\sqrt{n}}\right)$
distribution of sample mean $\bar{X}$	mean $\mathrm{E}(\bar{X}) = \mu$ variance $\mathrm{var}(\bar{X}) = \frac{\sigma^2}{n}$

Calculus

$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n \, dx = \frac{1}{n+1}x^{n+1} + c, \; n \neq -1$
$\frac{d}{dx}(\mathrm{e}^{ax}) = a\mathrm{e}^{ax}$	$\int \mathrm{e}^{ax} \, dx = \frac{1}{a}\mathrm{e}^{ax} + c$
$\frac{d}{dx}(\log_\mathrm{e}(x)) = \frac{1}{x}$	$\int \frac{1}{x} \, dx = \log_\mathrm{e} \lvert x \rvert + c$
$\frac{d}{dx}(\sin(ax)) = a\cos(ax)$	$\int \sin(ax) \, dx = -\frac{1}{a}\cos(ax) + c$
$\frac{d}{dx}(\cos(ax)) = -a\sin(ax)$	$\int \cos(ax) \, dx = \frac{1}{a}\sin(ax) + c$
$\frac{d}{dx}(\tan(ax)) = a\sec^2(ax)$	$\int \sec^2(ax) \, dx = \frac{1}{a}\tan(ax) + c$
$\frac{d}{dx}\left(\sin^{-1}(x)\right) = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{1}{\sqrt{a^2-x^2}} \, dx = \sin^{-1}\left(\frac{x}{a}\right) + c, \; a > 0$
$\frac{d}{dx}\left(\cos^{-1}(x)\right) = \frac{-1}{\sqrt{1-x^2}}$	$\int \frac{-1}{\sqrt{a^2-x^2}} \, dx = \cos^{-1}\left(\frac{x}{a}\right) + c, \; a > 0$
$\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$	$\int \frac{a}{a^2+x^2} \, dx = \tan^{-1}\left(\frac{x}{a}\right) + c$
	$\int (ax+b)^n \, dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, \; n \neq -1$
	$\int (ax+b)^{-1} \, dx = \frac{1}{a}\log_\mathrm{e} \lvert ax+b \rvert + c$
produce rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
chain rule	$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$
Euler's method	If $\frac{dy}{dx} = f(x)$, $x_0 = a$ and $y_0 = b$, then $x_{n+1} = x_n + h$ and $y_{n+1} = y_n + hf(x_n)$
acceleration	$a = \frac{d^2x}{dt^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$
arc length	$\int_{x_1}^{x_2} \sqrt{1 + (f'(x))^2} \, dx$ or $\int_{t_1}^{t_2} \sqrt{(x'(t))^2 + (y'(t))^2} \, dt$

Vectors in two and three dimensions

$\underset{\sim}{r} = x\underset{\sim}{i} + \underset{\sim}{j} + z\underset{\sim}{k}$
$\lvert \underset{\sim}{r} \rvert = \sqrt{x^2 + y^2 + z^2} = r$
$\dot{\underset{\sim}{r}} = \frac{d\underset{\sim}{r}}{dt} = \frac{dx}{dt}\underset{\sim}{i} + \frac{dy}{dt}\underset{\sim}{j} + \frac{dz}{dt}\underset{\sim}{k}$
$\underset{\sim}{r}_1 \cdot \underset{\sim}{r}_2 = r_1 r_2 \cos(\theta) = x_1x_2 + y_1y_2 + z_1z_2$

Mechanics

momentum	$\underset{\sim}{p} = m\underset{\sim}{v}$
equation of motion	$\underset{\sim}{R} = m\underset{\sim}{a}$

END OF FORMULA SHEET